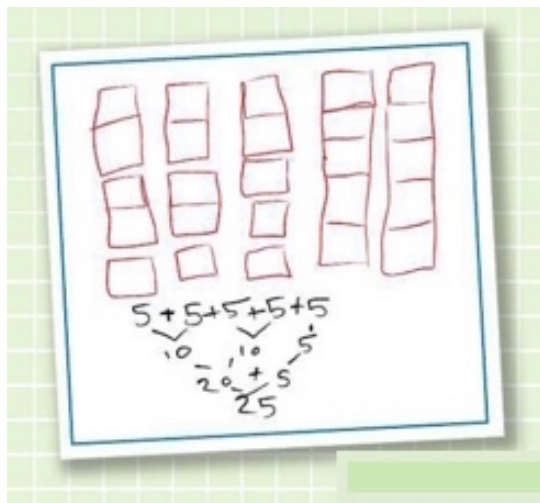
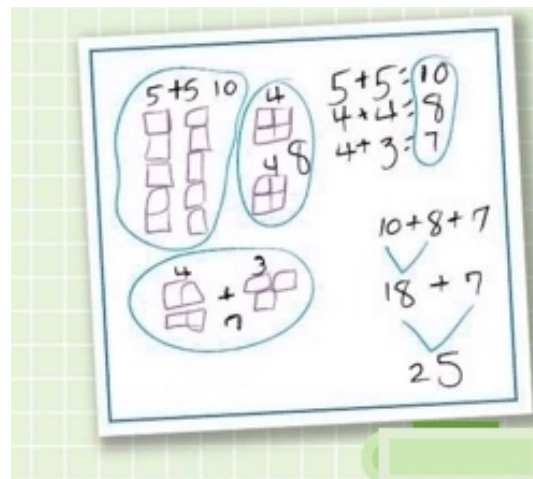


A



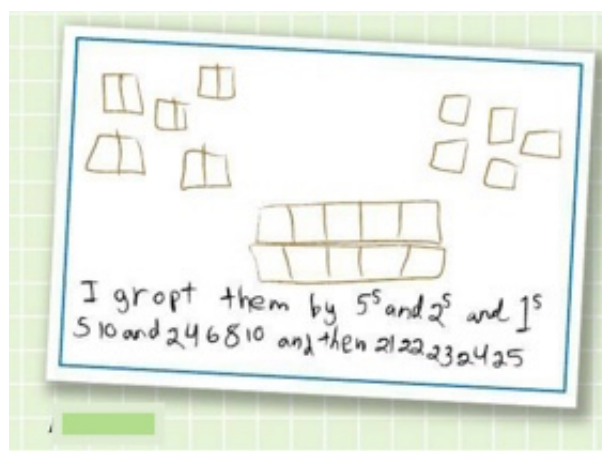
B



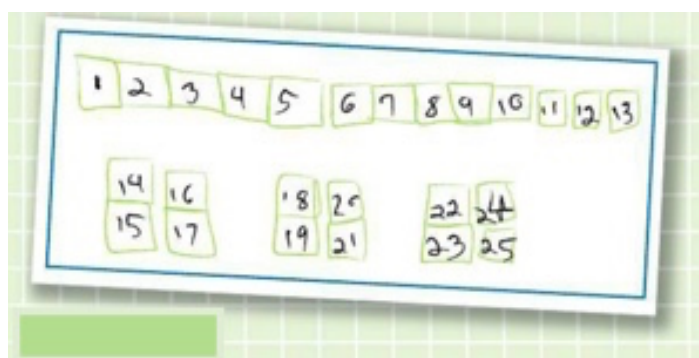
C



D



E



F



How Students Counted Squares

1 Bruce lays the squares out on his desk. He puts the single squares together and then carefully draws a copy of his arrangements. He counts the squares in his drawing starting with the singles, recording the counting number on the picture of the square after he counts it. He tells the teacher that he counted the squares in the drawing (Rather than the actual squares) because once he drew on a square he knows he has already counted it.

2 Sacha says, "I am going to count by 4's" She organizes the pairs of two and the single square into three groups of 4 and puts the remaining single squares in a row. She points to one group of 4 and says "4. Points to another and says "6... mm 6 and 8", points to the third group hesitates and then quickly says "10." After a pause she says "I think I'll try a different way."

This time she counted each square in the rows of five and then each square in the groups by 1's. She copies her arrangement on paper and correctly records the order in which she counted.

3 Tamika puts the two 5's together and organizes the rest of the squares from three groups of 4 and the group of 4. She draws her arrangement and circles the two groups of 5, two groups of 4 and the two remaining groups a 4 and a 3.

She records the number in each group, the number in each circle and the equations that corresponds to each circled group. She works quickly and appears to be using knowledge of number combinations to find the total. Next Tamika writes the expression in which the totals in her three equations are added $10 + 8 + 7$. She combines 10 and 8 and gets her next expression $10 + 7$. She arrives at a total of 25 by counting up 7 from 18. She kept track on her fingers as she counts "21, 22, 23, 24, 25."

4 Leah puts the strips of 5 in one corner, the pairs in another and the singles in the third. She points to each strip of 5, saying "5, 10." Next she counts the pairs touching each as she counts "2, 4, 6, 8, 10." She says, "Two 10's are 20" and then counts the single squares as she touched each square as she counts 21, 22, 23, 24, 25."

When the teacher asked Leah if she could count the squares a different way, she hesitates and then says that she would count each square. The teacher asked whether there might be yet a different way to count the squares, Leah seemed puzzled at first but after moving the squares around for a moment begins to organize them in groups of five.

5 Jacob arranged his squares in groups of 5. He recorded numbers under the cube recording $5 + 5 + 5 = 5 + 5$ and then grouped the fives into 10, 10 and 5. Next he grouped the 10's into twenty and added the remaining 5 to make twenty-five.

6 Jacinta did not organize the paper squares. She understood that she was to make a picture with squares and count them. She draws ten boxes in a row on her paper and ran out of space. She told her teacher that she was going to count the squares by two's. As she counted she wrote a number in a box and said the number out loud. She says, "2, 4, 6, 8, 10, 21, 23, 24, 26, 28."